Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 5
Due Wednesday February 20

12. (a) If b is a 99 digit number how many bits are needed to represent it? That is, if b is a 99 digit number, how many digits are needed when it is represented base 2? Your answer will be a range of several numbers. Consider how log (common logarithm, base 10) and lg (logarithm base 2) relate to the number of digits. Use this and basic facts about logarithms. We have not discussed basic rules for logarithm manipulation in class. If you do not recall these use any inanimate source that you like.
(b) Answer as in part (a) except for a $t-1$ digit number. Your answer should be a range of numbers specified by two values written in terms of t and some logarithms.

13. What is the smallest k such that the Fibonacci number F_k has at least 99 digits? What does this tell you about the number of steps in the Euclidean algorithm in the worst case if the smaller of the two numbers for which you determine the gcd has 99 digits? Recall that F_k is the integer closest to $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^k$. As in the previous problem, think about how the number of digits relates to the common logarithm and find and use some basic facts about logarithms.

14. Prove that for positive integers $a_1, a_2, \ldots, a_k, c$ we have that $a_1x_1 + a_2x_2 + \cdots + a_kx_k = c$ has a an integer solution only if c is a multiple of the greatest common divisor $\gcd(a_1, \ldots, a_k)$ of the a_i.

15. Prove that for positive integers $a_1, a_2, \ldots, a_k, c$ we have that $a_1x_1 + a_2x_2 + \cdots + a_kx_k = c$ has a an integer solution if c is a multiple of the greatest common divisor $\gcd(a_1, \ldots, a_k)$ of the a_i. Note that it is enough to show that there is a a solution when $c = \gcd(a_1, \ldots, a_k)$ and then use induction on k. You may use the $k = 2$ case proved in class as a basis. You may also use the fact that $\gcd(\gcd(a_1, \ldots, a_{k-1}), a_k) = \gcd(a_1, \ldots, a_k)$.

16. Consider the statement that exactly one of the following holds for given integers: $a_1, a_2, \ldots, a_k, c$: (I) $a_1x_1 + \cdots + a_kx_k = c$ has an integer solution $x_1, x_2, \ldots, x_k$; (II) ya_i integral for $i = 1, 2, \ldots, k$ and yc non-integral has a solution y.
Prove directly that at most one of (I) or (II) holds.

17. Consider the statement that exactly one of the following holds for given integers: $a_1, a_2, \ldots, a_k, c$: (I) $a_1x_1 + \cdots + a_kx_k = c$ has an integer solution $x_1, x_2, \ldots, x_k$; (II) ya_i integral for $i = 1, 2, \ldots, k$ and yc non-integral has a solution y.
This is really just a restatement of 14 and 15 above. Show this statement using those results. By 16 it is enough to show that at least one holds. Consider two cases, whether or not c is a multiple of $\gcd(a_1, \ldots, a_k)$ and explain why (using 14 or 15) this gives a solution in (I) or (II).