Lehigh University Calculus Diagnostic
August 21, 2009

Name_____

Write your answers here as well as on the answer sheet.
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Add or subtract terms whenever possible.

1) $\sqrt{3x} - 3\sqrt{108x} - 5\sqrt{27x}$ 1) _____

    A) $-8\sqrt{138x}$ B) $-32\sqrt{3x}$ C) $-32\sqrt{138x}$ D) $-8\sqrt{3x}$

Factor and simplify the algebraic expression.

2) $(x + 3)^{2/5} - (x + 3)^{12/5}$ 2) _____

    A) $(x + 3)^{2/5}(-x^2 - 6x - 8)$ B) $(x + 3)((x + 3)^{2/5} - (x + 3)^{12/5})$

    C) $(x + 3)(-x^2 - 6x + 8)$ D) $(x + 3)^{12/5}((x + 3)^{1/6} - 1)$

Multiply as indicated.

3) $\dfrac{x^2 + 9x + 20}{x^2 + x - 20} \cdot \dfrac{x^2 - 25}{x^2 - x - 20}$ 3) _____

    A) $\dfrac{x - 5}{x - 4}$ B) $\dfrac{x + 4}{x - 5}$ C) $\dfrac{x + 5}{x - 4}$ D) $\dfrac{x - 5}{x + 4}$

Simplify the expression.

4) $\dfrac{\sqrt{x} - \dfrac{1}{2\sqrt{x}}}{\sqrt{x}}$ 4) _____

    A) $1 - \dfrac{1}{2}$ B) $\dfrac{\sqrt{x} - \dfrac{1}{2\sqrt{x}}}{\sqrt{x}}$ C) $x^2 - \dfrac{1}{2x}$ D) $1 - \dfrac{1}{2x}$

Solve the formula for the specified variable.

5) $P = \dfrac{A}{1 + rt}$ for r   5) _____

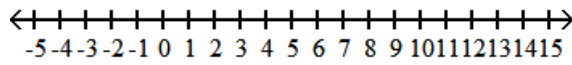   A) $r = \dfrac{P - 1}{At}$ B) $r = \dfrac{P - A}{1 + t}$ C) $r = P - At$ D) $r = \dfrac{A - P}{Pt}$

Solve the quadratic equation using the quadratic formula.

6) $3x^2 = -12x - 2$ 6) _____

    A) $\left\{ \dfrac{-6 - \sqrt{30}}{3}, \dfrac{-6 + \sqrt{30}}{3} \right\}$ B) $\left\{ \dfrac{-6 - \sqrt{42}}{3}, \dfrac{-6 + \sqrt{42}}{3} \right\}$

    C) $\left\{ \dfrac{-6 - \sqrt{30}}{6}, \dfrac{-6 + \sqrt{30}}{6} \right\}$ D) $\left\{ \dfrac{-12 - \sqrt{30}}{3}, \dfrac{-12 + \sqrt{30}}{3} \right\}$

Solve the compound inequality. Other than ∅, use interval notation to express the solution set and graph the solution set on a number line.

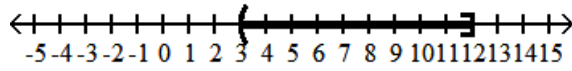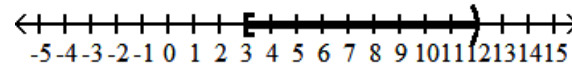7) $10 \leq \frac{2}{3}x + 8 < 16$

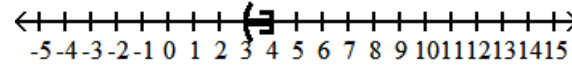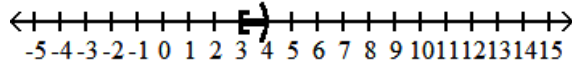7) _____

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  -5-4-3-2-1 0 1 2 3 4 5 6 7 8 9 1011112131415
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A) (3, 12]

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  -5-4-3-2-1 0 1 2 3 4 5 6 7 8 9 1011112131415
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B) [3, 12)

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C) [3, 4)

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D) (3, 4]

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  -5-4-3-2-1 0 1 2 3 4 5 6 7 8 9 1011112131415
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Evaluate the function at the given value of the independent variable and simplify.

8) $f(x) = 4x^2 + 2x + 6$; $f(x - 1)$

8) _____

A) $-6x^2 + 4x + 8$ B) $4x^2 + 26x + 12$ C) $4x^2 - 6x + 12$ D) $4x^2 - 6x + 8$

Find and simplify the difference quotient $\frac{f(x + h) - f(x)}{h}$, $h \neq 0$ for the given function.

9) $f(x) = \frac{1}{2x}$

9) _____

A) $\frac{-1}{x(x + h)}$ B) $\frac{-1}{2x(x + h)}$ C) $\frac{1}{2x}$ D) 0

Use the given conditions to write an equation for the line.

10) Passing through (2, 5) and (4, 2)

10) _____

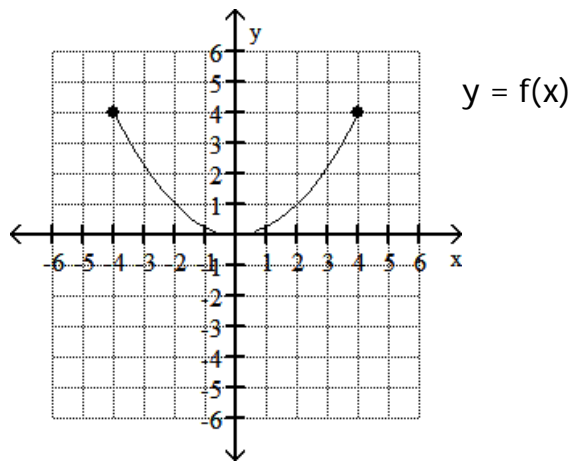A) $y = -\frac{3}{2}x + 8$ B) $y = mx + 8$ C) $y = -\frac{3}{2}x - 2$ D) $y = \frac{3}{2}x + 8$
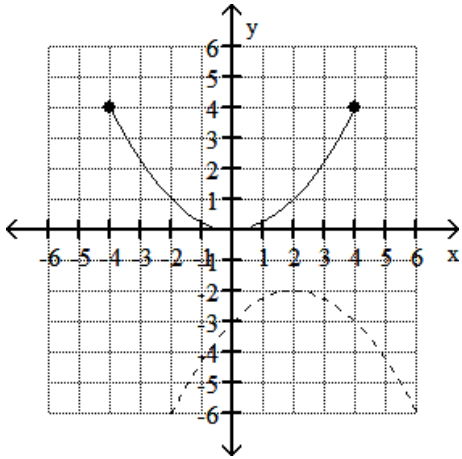
2

Use the graph of the function f, plotted with a solid line, to sketch the graph of the given function g.
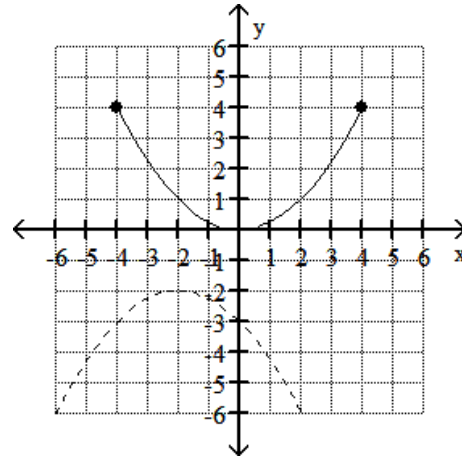
11) g(x) = -f(x - 2) - 2

11) _____

y = f(x)
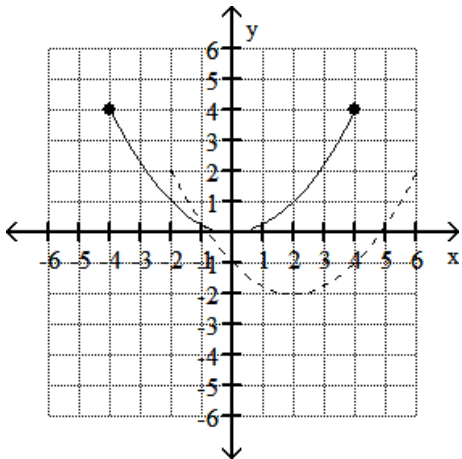
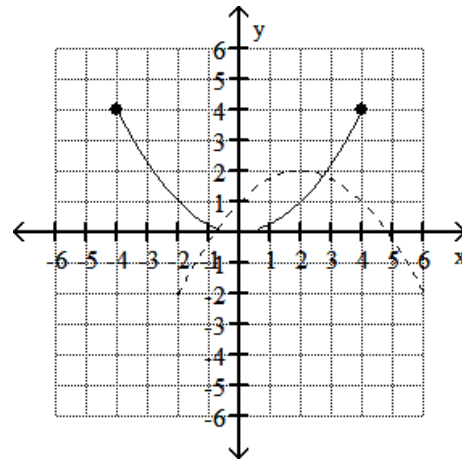A)

B)

C)

D)

Find functions f and g so that h(x) = (f ∘ g)(x).

12) $h(x) = \dfrac{6}{\sqrt{8x + 8}}$

12) _____

A) $f(x) = 6,\ g(x) = \sqrt{8x + 8}$

B) $f(x) = 6/x,\ g(x) = 8x + 8$

C) $f(x) = 6/\sqrt{x},\ g(x) = 8x + 8$

D) $f(x) = \sqrt{8x + 8},\ g(x) = 6$

Find the inverse of the one-to-one function.

13) $f(x) = \dfrac{2x + 5}{7}$

13) _____

A) $f^{-1}(x) = \dfrac{7}{2x + 5}$

B) $f^{-1}(x) = \dfrac{7x - 5}{2}$

C) $f^{-1}(x) = \dfrac{7}{2x - 5}$

D) $f^{-1}(x) = \dfrac{7x + 5}{2}$

3

Solve the problem.

14) The annual yield per apple tree is fairly constant at 270 pounds per tree when the number of   14) _____
trees per acre is 50 or fewer. For each additional tree over 50, the annual yield per tree for all
trees on the acre decreases by 3 pounds due to overcrowding. For $x > 50$, express the yield per
tree, Y, in pounds, as a function of the number of apple trees per acre, x.

    A) $Y(x) = -3x + 420$ B) $Y(x) = 3x + 120$
    C) $Y(x) = -3x + 120$ D) $Y(x) = 3x + 420$

Divide.

15) $\dfrac{x^4 + 16}{x - 2}$ 15) _____

    A) $x^3 + 2x^2 + 4x + 8 + \dfrac{16}{x - 2}$ B) $x^3 + 2x^2 + 4x + 8 + \dfrac{32}{x - 2}$

    C) $x^3 + 2x^2 + 4x + 8$ D) $x^3 - 2x^2 + 4x - 8 + \dfrac{32}{x - 2}$

Find a rational zero of the polynomial function and use it to find all the zeros of the function.

16) $f(x) = 3x^3 - x^2 - 9x + 3$ 16) _____
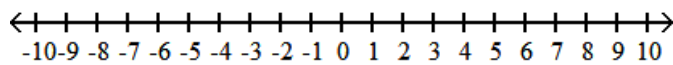
    A) $\{\frac{1}{3}, 3, -3\}$ B) $\{3, 1+\sqrt{3}, 1 - \sqrt{3}\}$ C) $\{3, \sqrt{3}, -\sqrt{3}\}$ D) $\{\frac{1}{3}, \sqrt{3}, -\sqrt{3}\}$
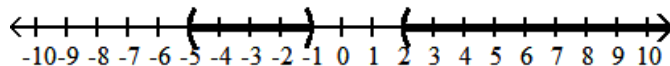
Solve the polynomial inequality and graph the solution set on a number line. Express the solution set in interval
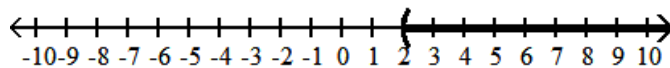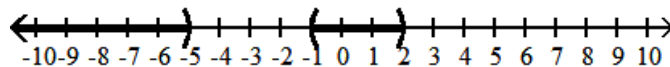notation.

17) $(x + 5)(x + 1)(x - 2) > 0$ 17) _____

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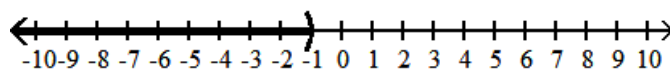   -10-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

    A) $(-5, -1) \cup (2, \infty)$

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    -10-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

    B) $(2, \infty)$

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    -10-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

    C) $(-\infty, -5) \cup (-1, 2)$

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    -10-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

    D) $(-\infty, -1)$

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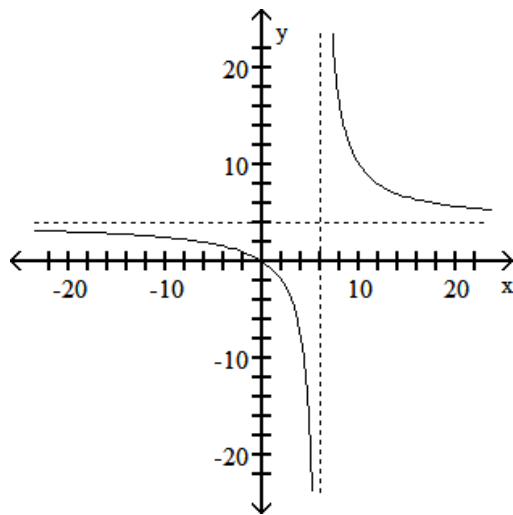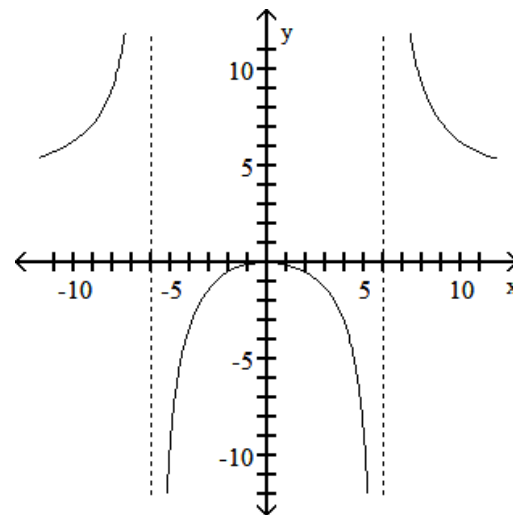   -10-9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

4

Graph the rational function.
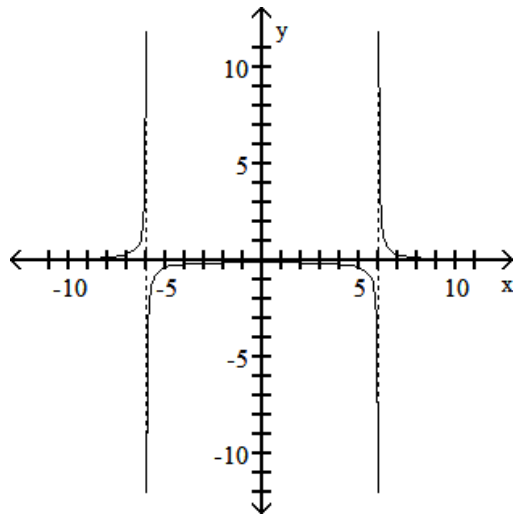
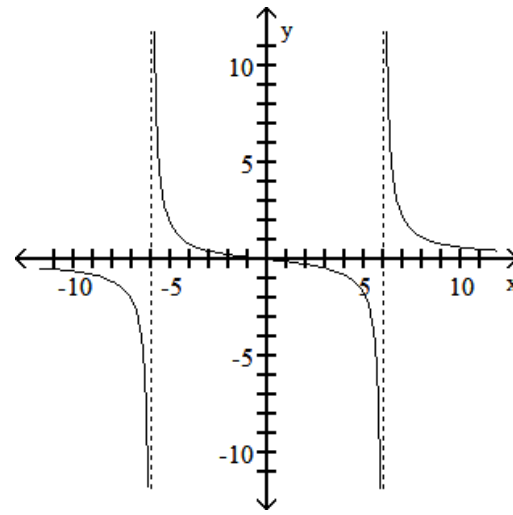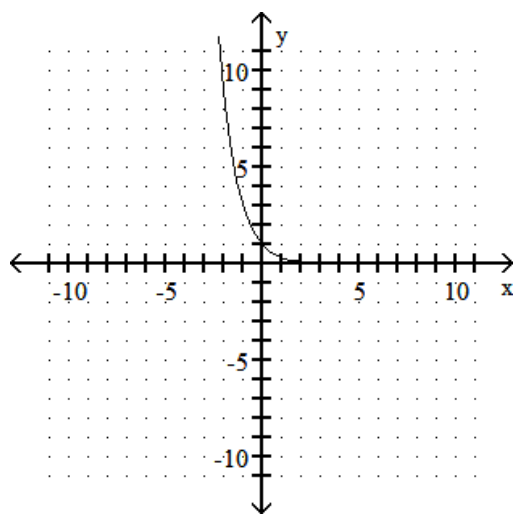18) $f(x) = \dfrac{4x}{x^2 - 36}$

18) _____

A)

B)

C)

D)

The graph of an exponential function is given. Select the function for the graph from the functions listed.

19)

19) _____

A) $f(x) = 3^x$
B) $f(x) = 3^{-x}$
C) $f(x) = -3^{-x}$
D) $f(x) = -3^x$

Use properties of logarithms to condense the logarithmic expression. Write the expression as a single logarithm.

20) $\frac{1}{6}(\log_7 x + \log_7 y) - 3\log_7 (x + 7)$

20) _____

A) $\log_7 \dfrac{\sqrt[6]{x} + \sqrt[6]{y}}{(x + 7)^3}$

B) $\log_7 \dfrac{\sqrt[6]{xy}}{3(x + 7)}$

C) $\log_7 \dfrac{\sqrt[6]{xy}}{(x + 7)^3}$

D) $\log_7 \dfrac{\sqrt[6]{x + y}}{(x + 7)^3}$

Solve the exponential equation. Express the solution set in terms of natural logarithms.

21) $8^{4x} = 3.4$

21) _____

A) $\left\{\dfrac{\ln 3.4}{8 \ln 4}\right\}$

B) $\left\{\dfrac{3.4 \ln 4}{\ln 8}\right\}$

C) $\left\{\dfrac{\ln 3.4}{4 \ln 8}\right\}$

D) $\left\{\dfrac{4 \ln 3.4}{\ln 8}\right\}$

The point P(x, y) on the unit circle that corresponds to a real number t is given. Find the value of the indicated trigonometric function at t.

22) $\left(\dfrac{5}{6}, -\dfrac{\sqrt{11}}{6}\right)$ Find csc t.

22) _____

A) $-\dfrac{6\sqrt{11}}{11}$

B) $-\dfrac{\sqrt{11}}{6}$

C) $\dfrac{\sqrt{11}}{5}$

D) $\dfrac{\sqrt{11}}{6}$

Use periodic properties of the trigonometric functions to find the exact value of the expression.

23) $\tan \dfrac{13\pi}{4}$

23) _____

A) -1

B) $\sqrt{3}$

C) $\dfrac{\sqrt{3}}{3}$

D) 1

Solve the problem.

24) A radio transmission tower is 120 feet tall. How long should a guy wire be if it is to be attached 10 feet from the top and is to make an angle of 45° with the ground? You may approximate cos 45° = sin 45° with .7. Give your answer to the nearest foot.
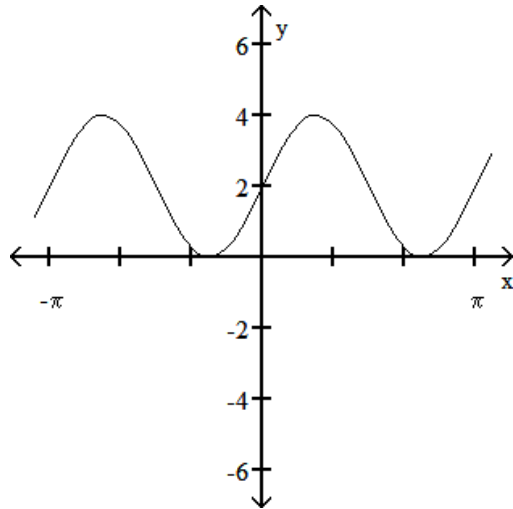
24) _____

A) 142 feet

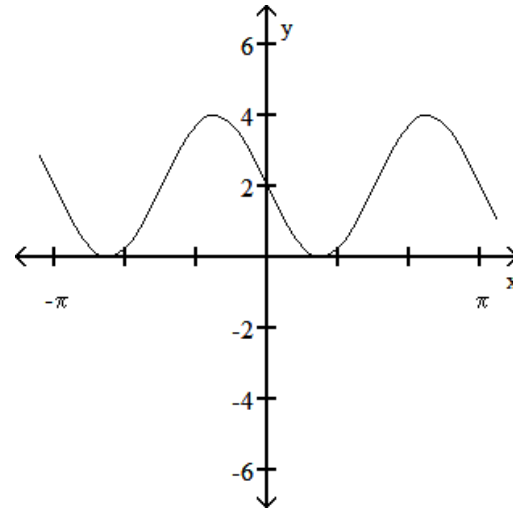B) 157 feet

C) 171 feet

D) 195 feet

Graph the function.

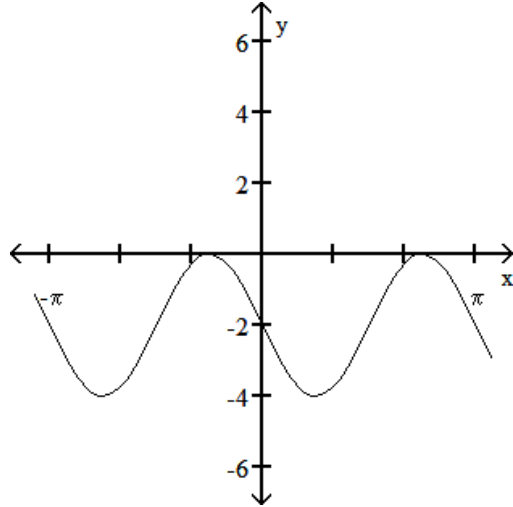25) $y = 2 \cos\left(2x - \dfrac{\pi}{2}\right) + 2$

A)

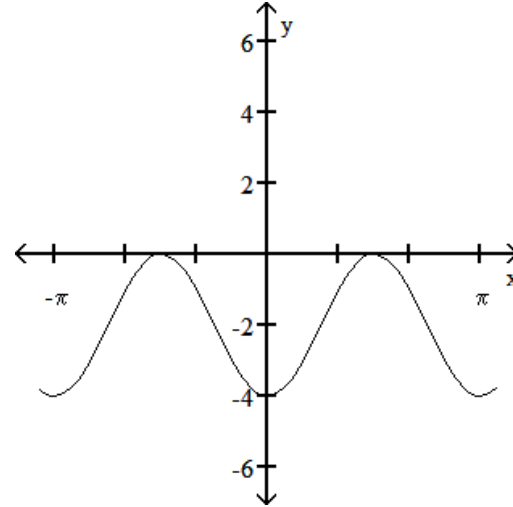B)

C)

D)

Use a right triangle to write the expression as an algebraic expression. Assume that x is positive and in the domain of the given inverse trigonometric function.

26) $\sin\left(\tan^{-1}\dfrac{x}{\sqrt{5}}\right)$

26) _____

A) $\dfrac{\sqrt{x^2 + 5}}{x^2 + 5}$

B) $\dfrac{x\sqrt{x^2 - 5}}{x^2 - 5}$

C) $\dfrac{x\sqrt{x^2 + 5}}{x^2 + 5}$

D) $x\sqrt{x^2 + 5}$

Complete the identity.

27) $1 - \dfrac{\sin^2 x}{1 + \cos x} = ?$

27) _____

A) $\cos x$

B) 0

C) $\cot x$

D) $\tan x$

Find all solutions of the equation.

28) $\tan x = \dfrac{\sqrt{3}}{3}$

28) _____

A) $x = \dfrac{2\pi}{3} + 2n\pi$

B) $x = \dfrac{5\pi}{6} + n\pi$

C) $x = \dfrac{\pi}{6} + n\pi$

D) $x = \dfrac{5\pi}{6} + 2n\pi$