Structure of Bipartite Probe Interval Graphs

David E. Brown Utah State University

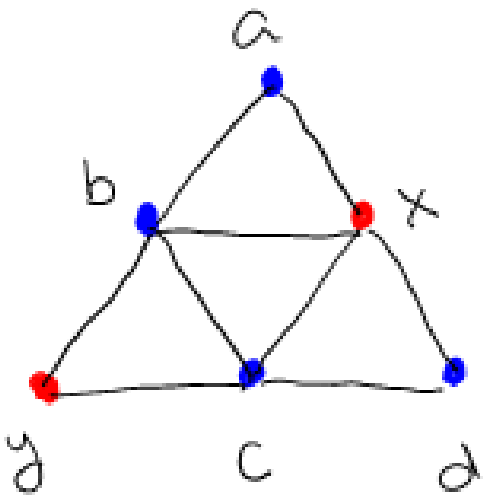
Arthur H. Busch University of Dayton

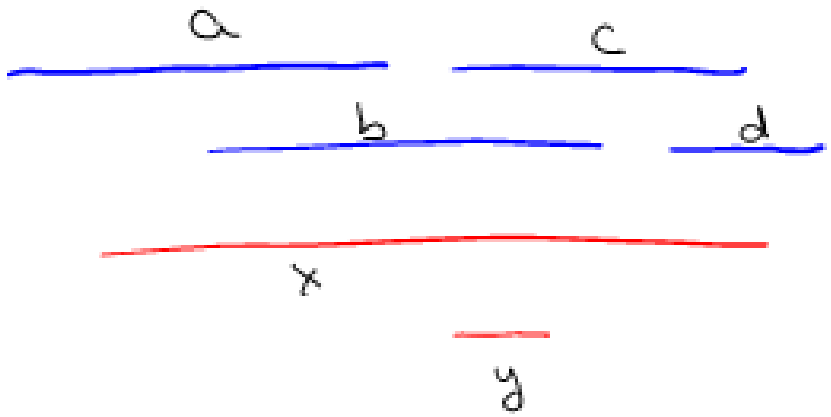
Garth Isaak Lehigh University

Outline

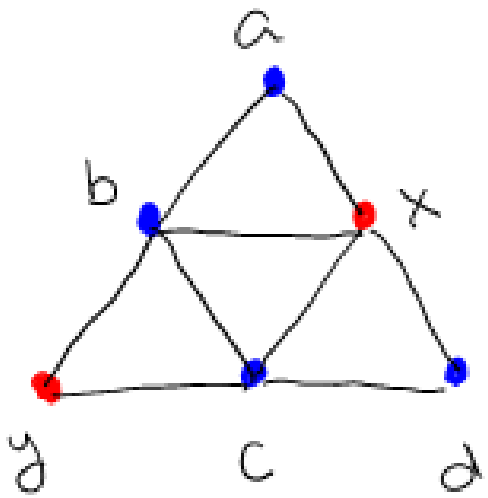
- (Bipartite) Probe Interval graphs
- **Some context and related problems**
- Structure of bipartite probe interval graphs

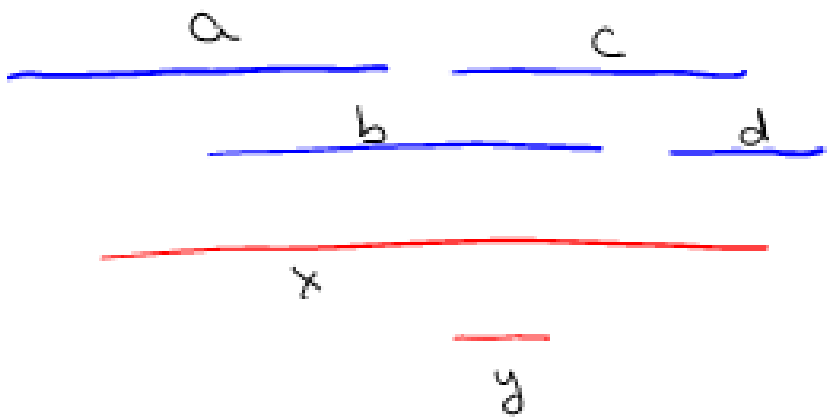
Probe Interval Graphs





Probe Interval Graphs

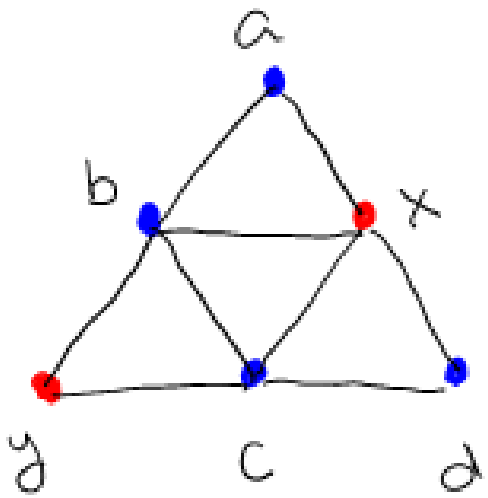


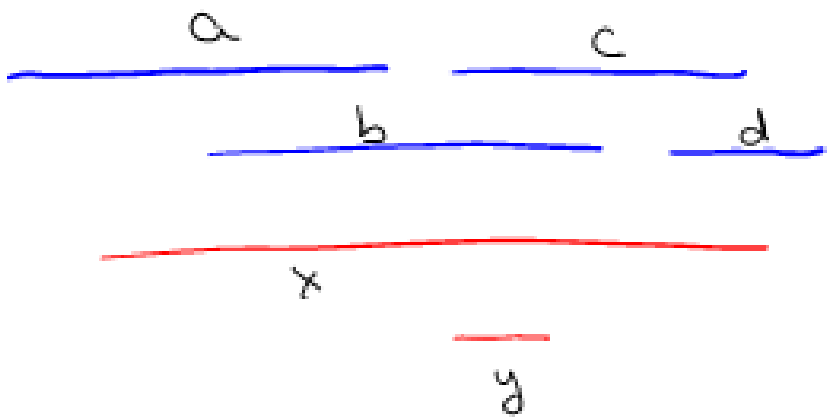


Intersection Graph of a set of Intervals

Record edges if at least one vertex is a probe

Probe Interval Graphs



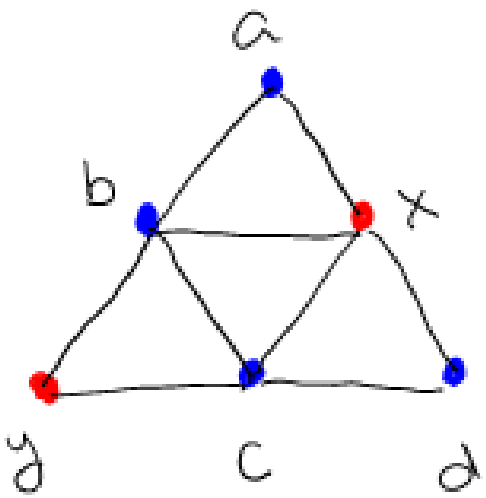


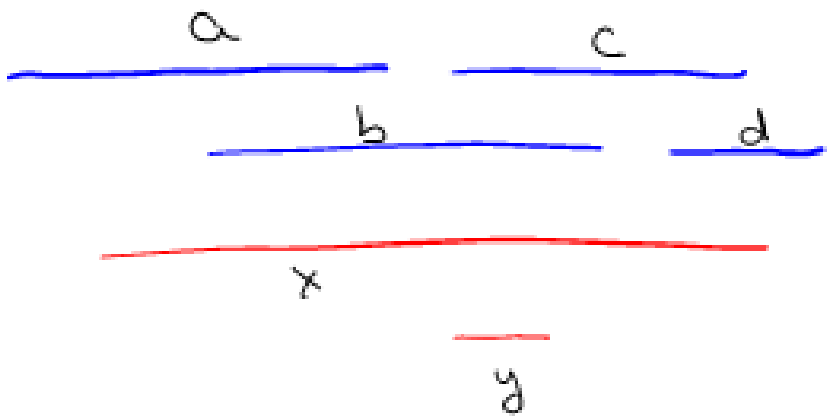
Intersection Graph of a set of Intervals

Record edges if at least one vertex is a probe

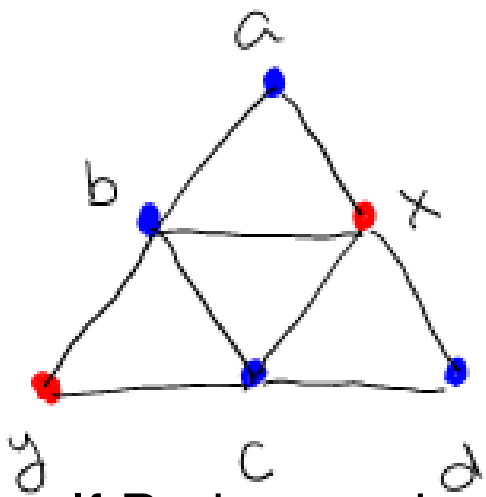
Introduced for an application in DNA sequencing - Zhang (1994)

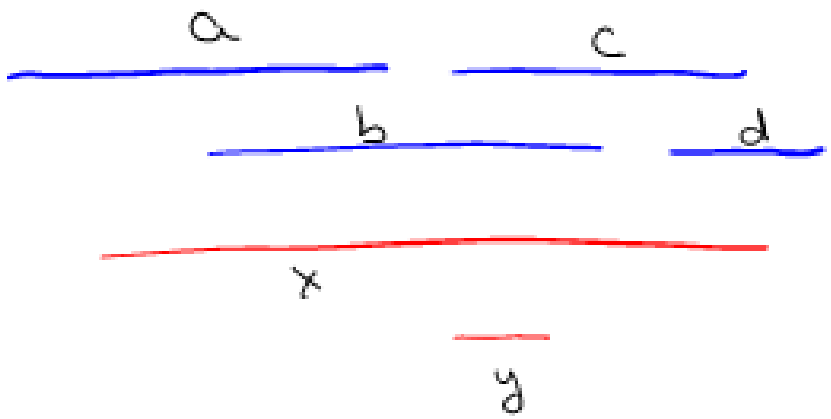
Does a given graph have a probe interval representation?





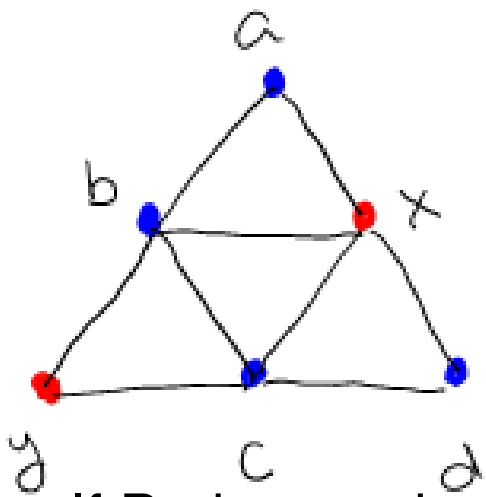
Does a given graph have a probe interval representation?

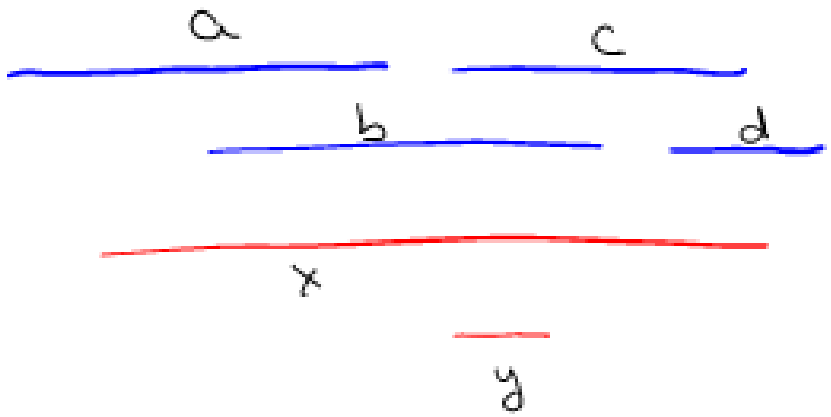




- If Probes and non-probes identified - $O(n + e \log n)$ recognition algorithm (Johnson and Spinrad (2001), McConnell and Spinrad (2002))

Does a given graph have a probe interval representation?





- If Probes and non-probes identified - $O(n + e \log n)$ recognition algorithm (Johnson and Spinrad (2001), McConnell and Spinrad (2002))
- If Probes and non-probes not identified - Polynomial recognition algorithm by Chang, Kloks, Liu, Peng (2005) - complexity?

Recognizing Probe Interval Graphs

Find a certifying Algorithm -

If ‘yes’ algorithm produces a representation

If ‘no’ algorithm produces an independent certificate that there is no probe representation

Recognizing Probe Interval Graphs

Find a certifying Algorithm -

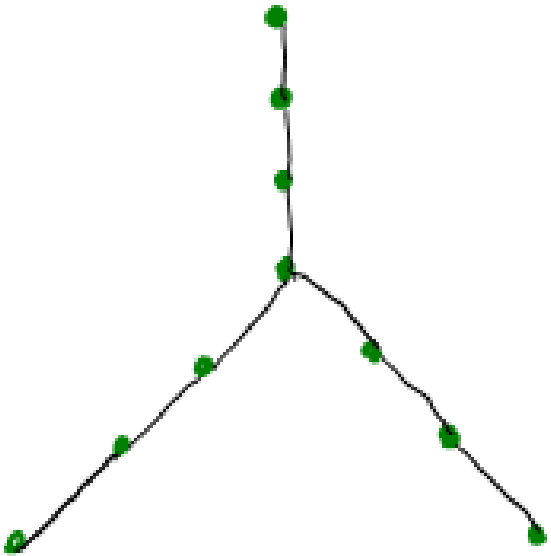
If ‘yes’ algorithm produces a representation

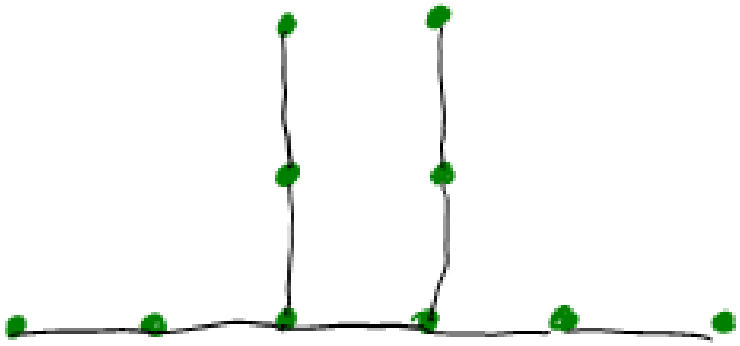
If ‘no’ algorithm produces an independent certificate that there is no probe representation

Need a characterization theorem

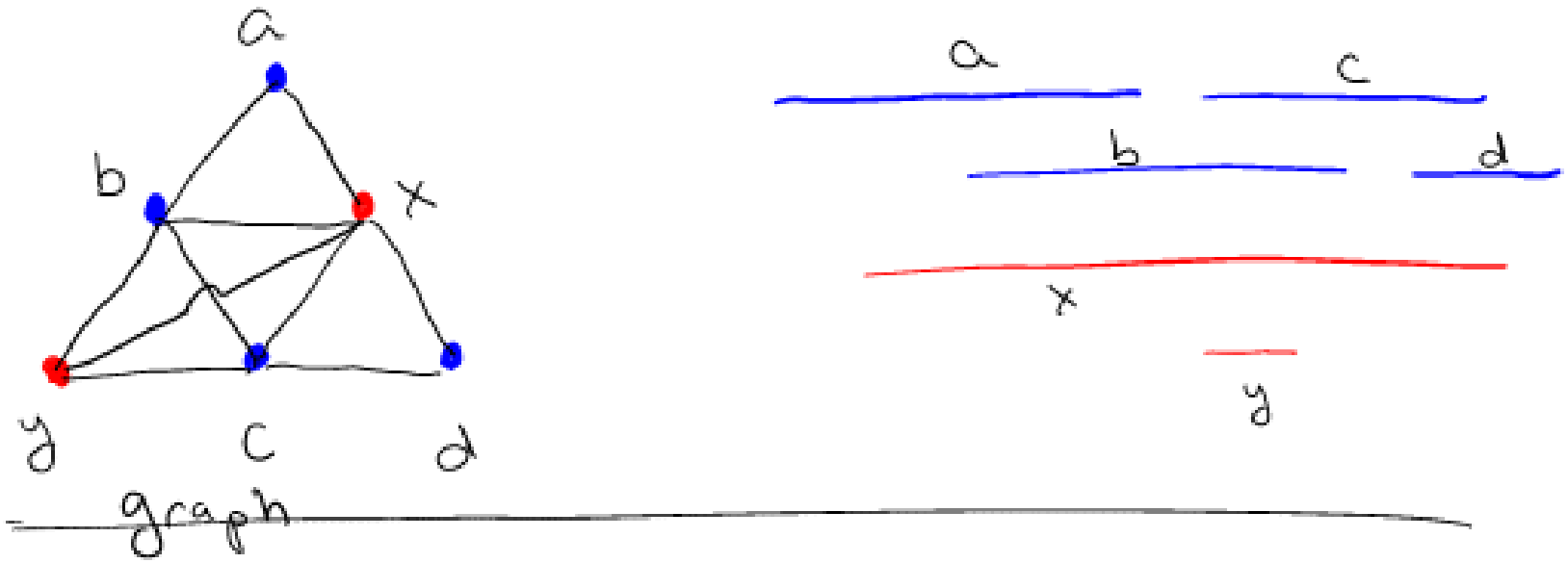
We will do this for bipartite graphs

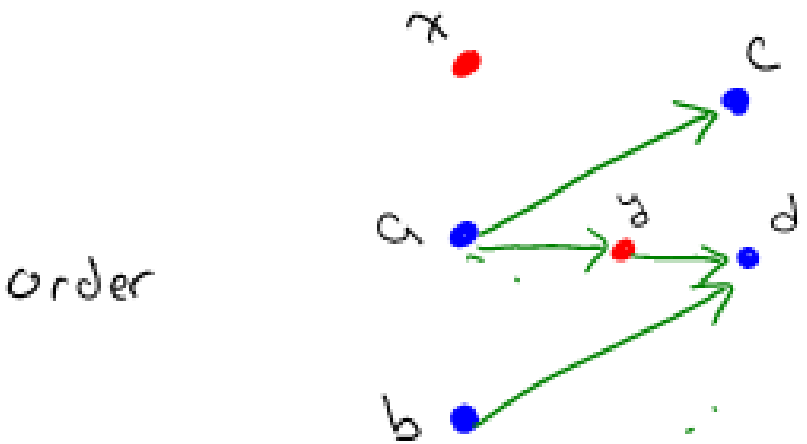
Sheng (1999) A tree is a probe interval graph if and only if it does not contain





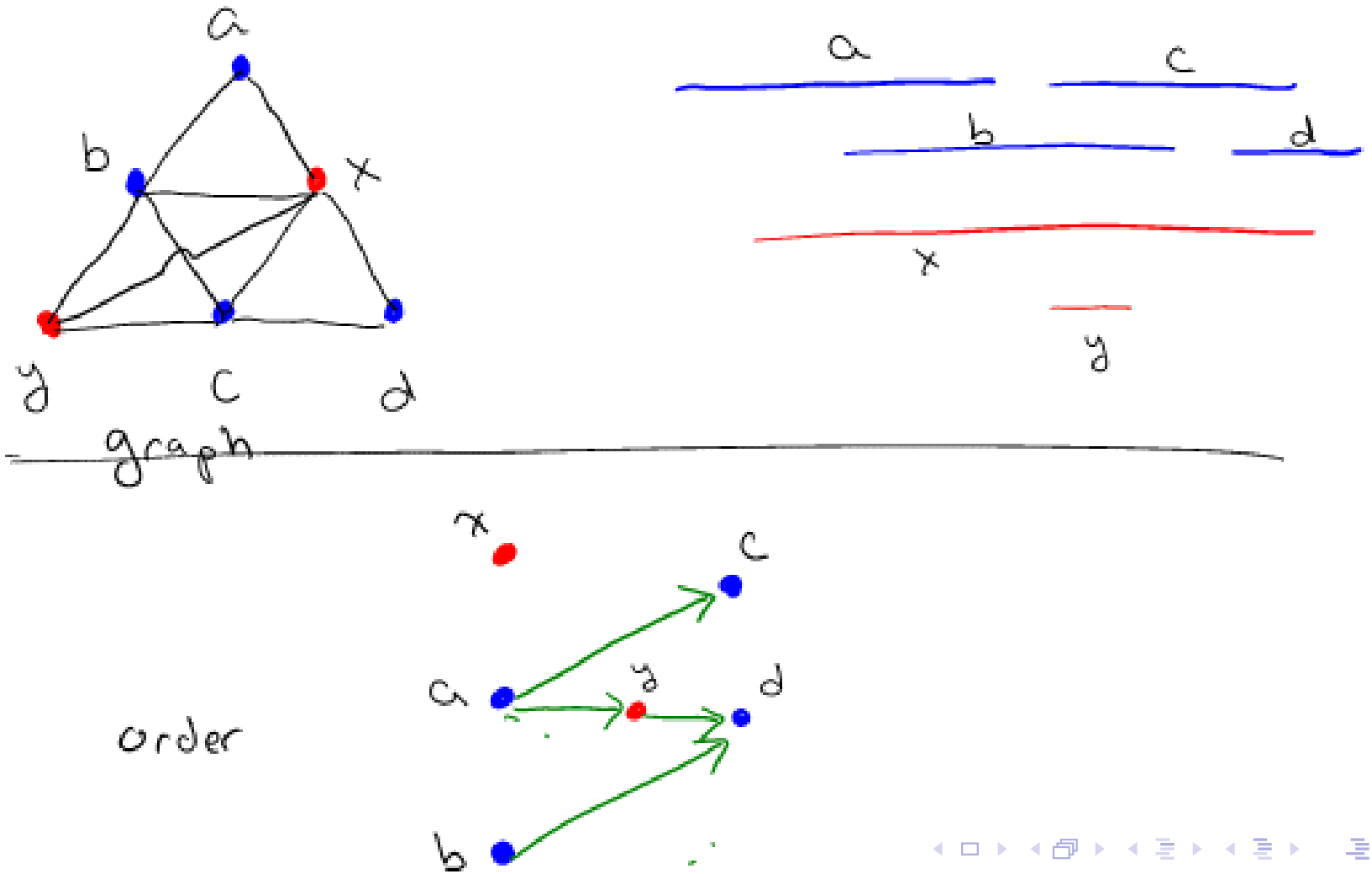
(finite) Interval graphs - intersection graphs of intervals





(finite) Interval graphs - intersection graphs of intervals

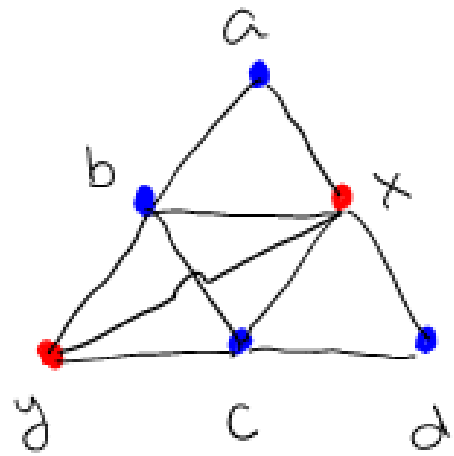
Probe interval graphs with all probes are interval graphs

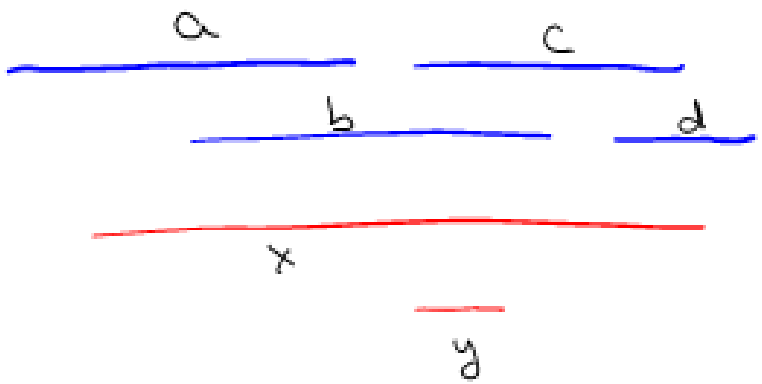


Interval Graphs are cocomparability

Complement has a transitive orientation

use ‘to the right of’





graph

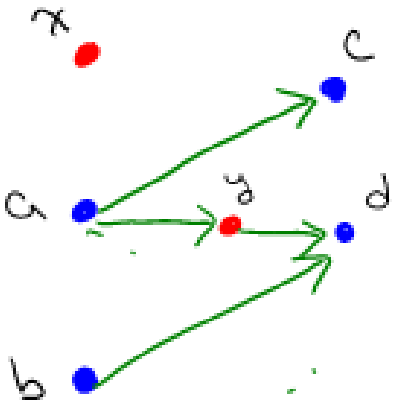
Ordered version

Weiner (1914) - time intervals

Fishburn (1970) - preference

relations

order



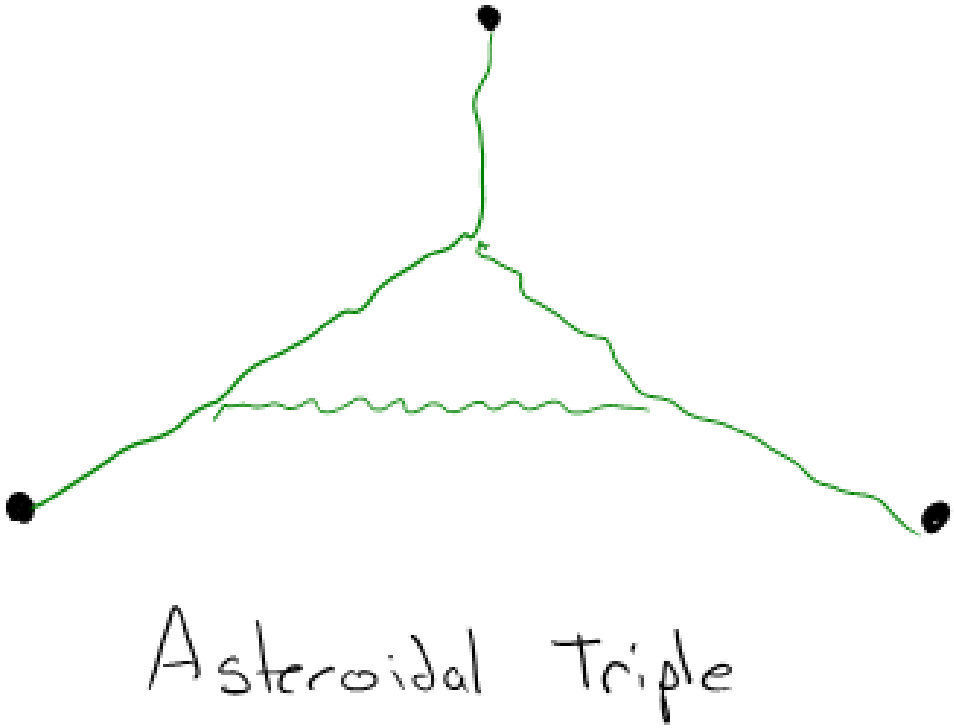
Interval graphs

- Hajos (1957)
- Benzer (1957) Gene arrangement on chromosones
- Gilmore and Hoffman (1964) G is interval graph if and only if it has no induced 4-cycle and is cocomparability
- Lekkerkerker and Boland (1962) G is an interval graph if and only if G is triangulated and asteroidal triple free.

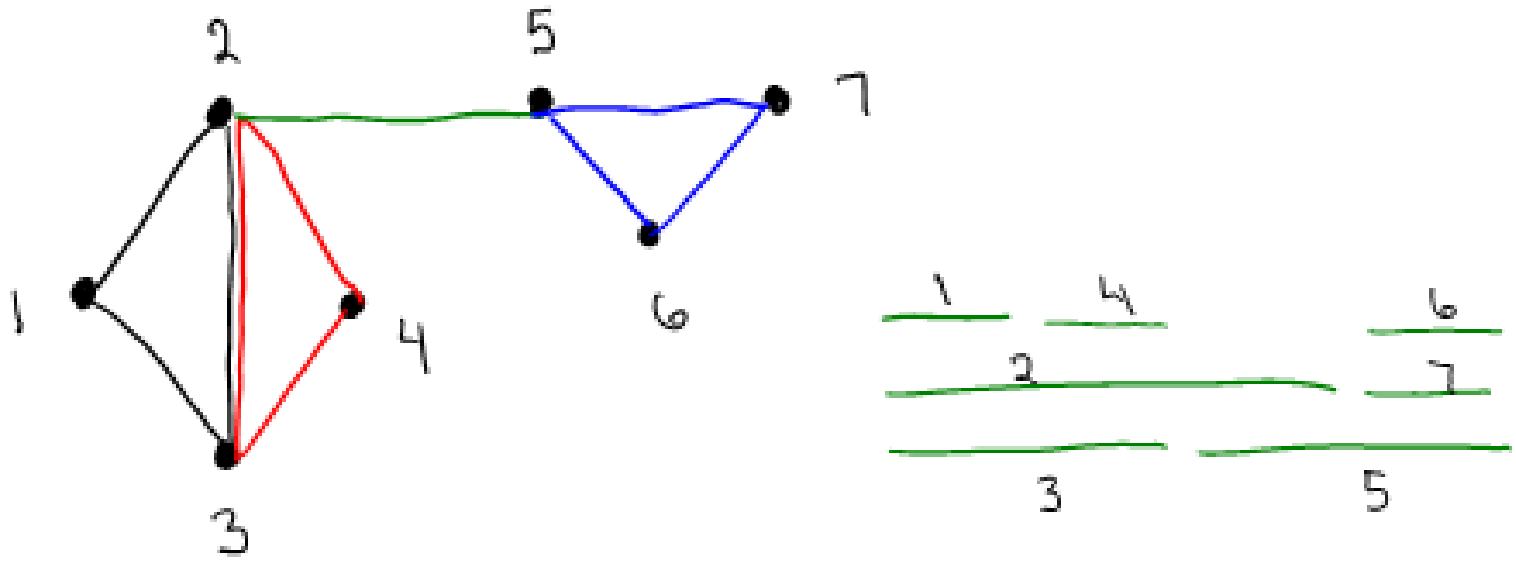
Gilmore and Hoffman (1964)
G is interval graph if and only if it has no induced 4-cycle and is cocomparability

Lekkerkerker and Boland (1962)

G is an interval graph if and only if *G* is triangulated and asteroidal triple free.



Lekkerkerker and Boland (1962)
G is an interval graph if and only if *G* there is a consecutive ordering of the maximal cliques - each vertex appears consecutively in this order



Ordering corresponds to right endpoints ...
Basis of a fast recognition algorithm by Booth and Leuker (1976) using PQ-trees

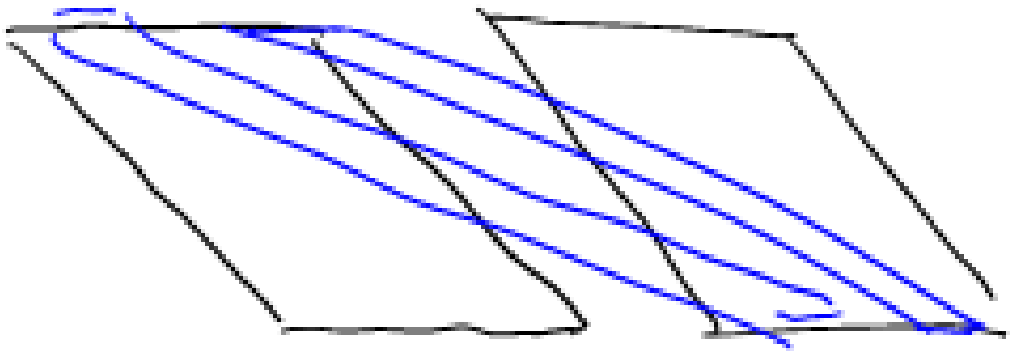
Tolerance interval graphs

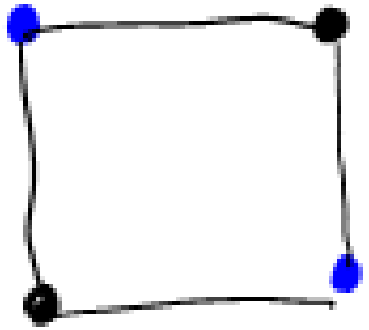
(one of many generalizations - other intersection models other versions of tolerance ...)

Golumbic and Monma (1982) and Golumbic, Monma and Trotter (1984)

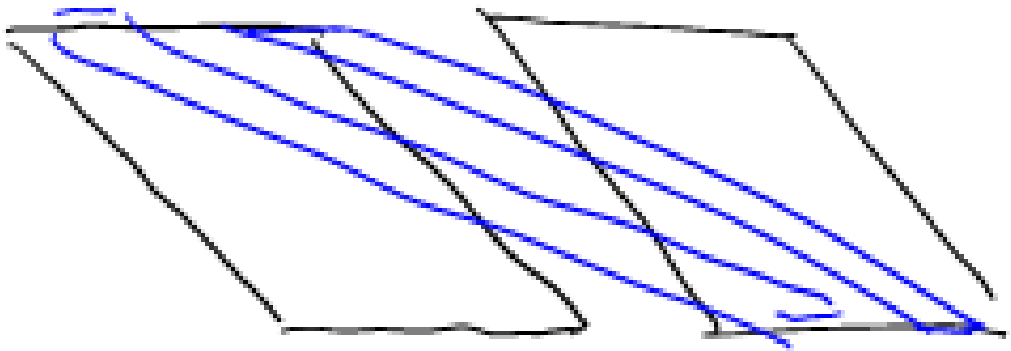
Intersection of intervals - record an edge only if intersection is 'big enough' (bigger than tolerance of at least one of the intervals) $xy \in E \Leftrightarrow |I_x \cap I_y| \geq \min\{t_x, t_y\}$

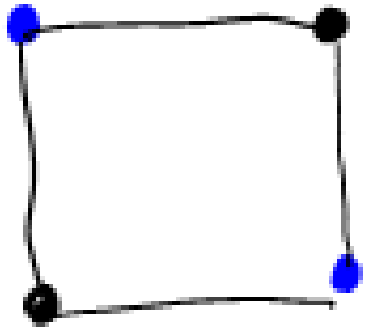
Tolerance graphs are intersection graphs of parallelograms ....





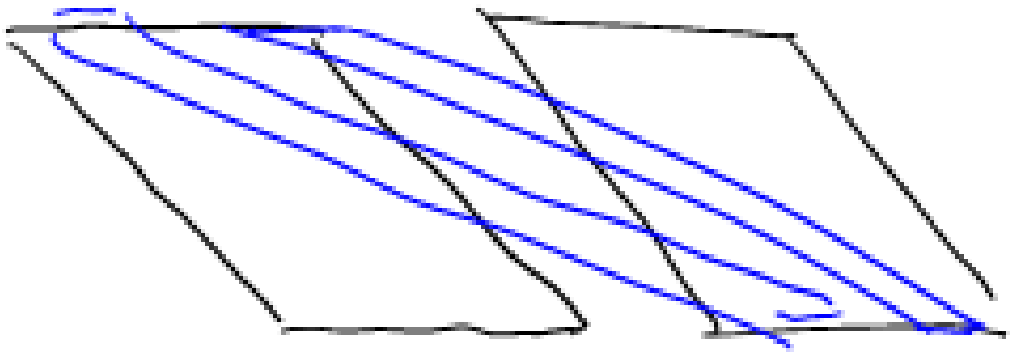
Bounded Tolerance graphs are intersection graphs of parallelograms ....

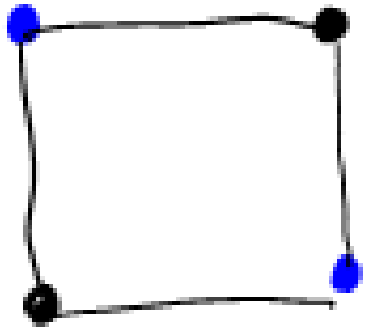




- Bounded - tolerances are finite

Bounded Tolerance graphs are intersection graphs of parallelograms ....





- Bounded - tolerances are finite

Open question from Golumbic, Monma, Trotter

- If G is cocomparability and tolerance is it bounded tolerance?

(Unbounded) Tolerance graphs

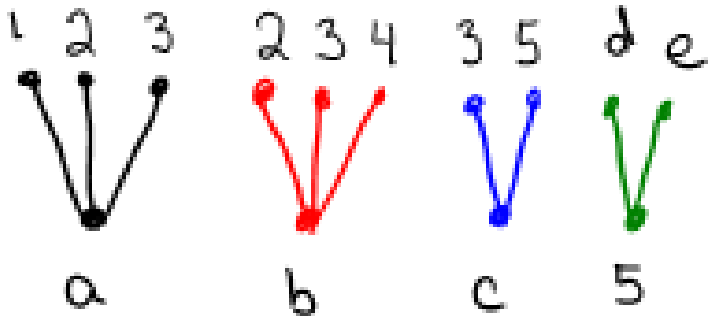
Probe interval graphs are tolerance graph where every tolerance is 0 or ∞.

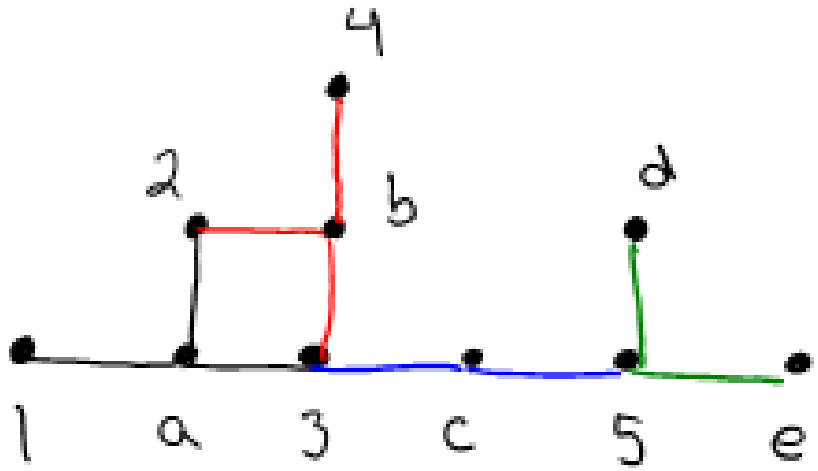
Characterization, recognition complexity etc for tolerance graphs are open.

Recognition and structure for bipartite tolerance graphs

Busch (2006) - G is a bipartite tolerance graph if and only if there is a consecutive star partition of the edges -

ordered sequence of stars partitioning edges such that each vertex appears consecutively.

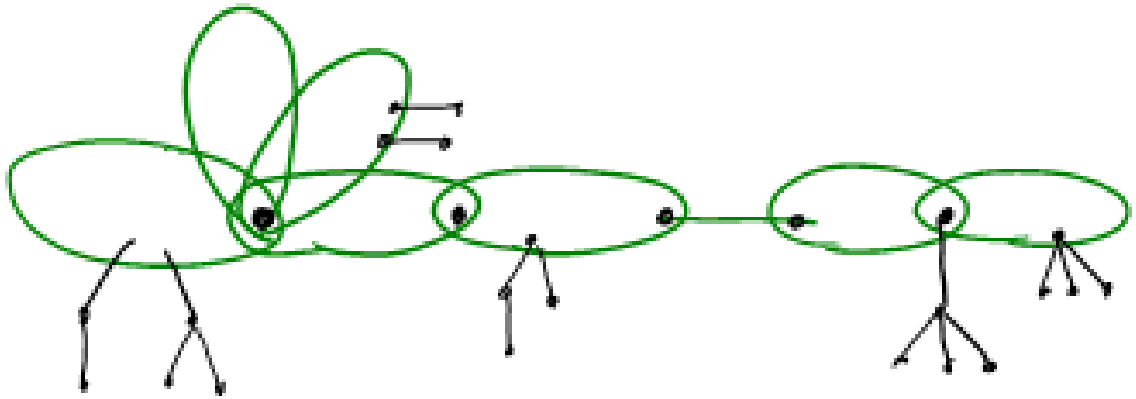


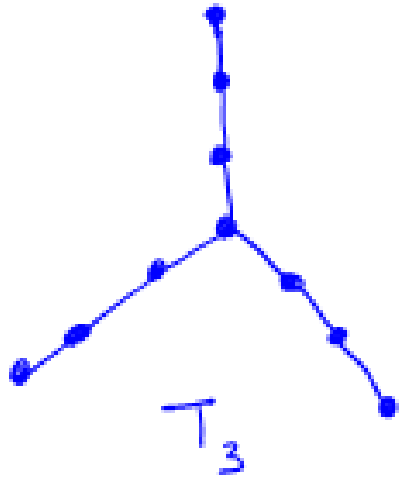


ideas similar to consecutive clique ordering for interval graphs

Recognition and structure for bipartite tolerance graphs

Busch and Isaak (2007) - Consecutive star partition ideas lead to linear recognition and certifying algorithm and structure theorem





'Theorem' - each 2-connected component (plus ...) is asteroidal triple free in at least one of the parts and no T_3.

Recognition and structure for bipartite probe interval graphs

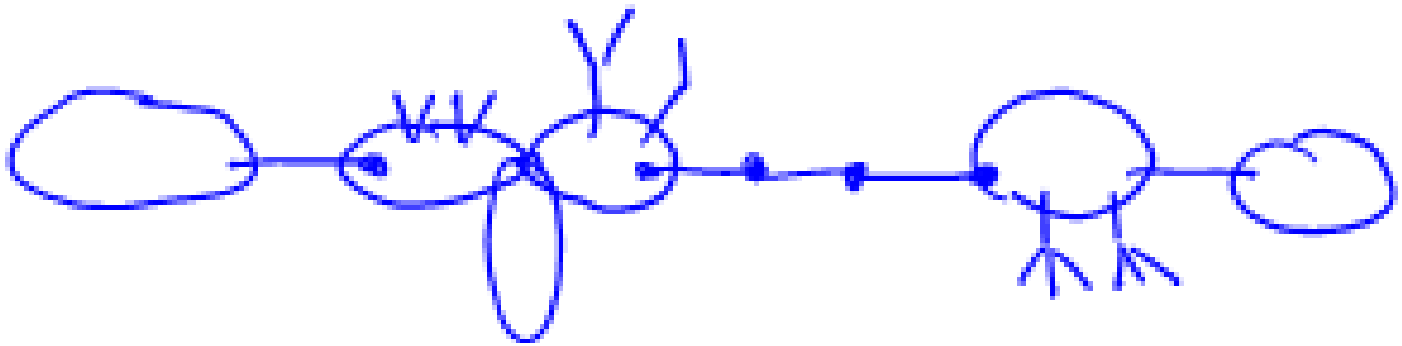
Brown (2006) G is a bipartite probe interval graph if and only if it has a consecutive U-star partition -

ordered sequence of stars partitioning edges such that each vertex appears consecutively (and star center switch parts on a star that is a single edge)

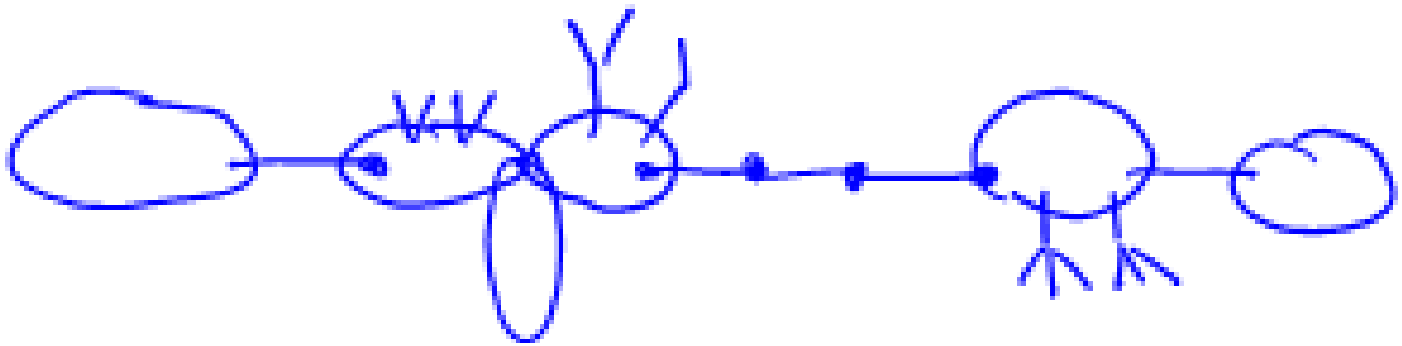
Brown, Busch and Isaak (2009) - Consecutive U star partition ideas lead to linear recognition and certifying algorithm and structure theorem for bipartite probe interval graphs

Each 2-edge connected component (plus ...) is asteroidal triple free in at least one of the parts and ....

Basic structure of bipartite probe interval graphs



Basic structure of bipartite probe interval graphs

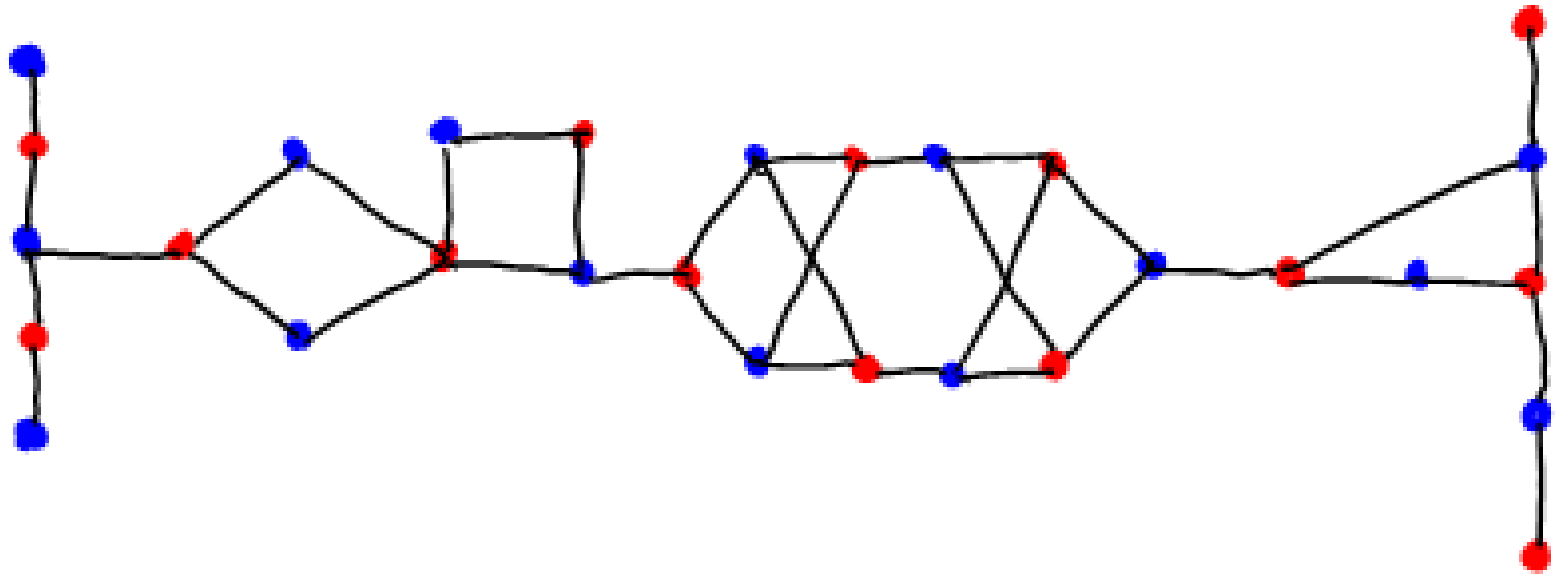


Two-edge connected components have a linear arrangement

Two-edge connected components (plus some dangling edges) are asteroidal triple free in at least one part

AND ... (something about cut edges ...)

Non bipartite probe example



bad because
blue asteroidal triple at left end
red asteroidal triple at right end
and all cut edges have blue on left